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TWO EVASION PROBLEMS IN A PLANE WITH SEPARATE CONTROLS AND NON-CONVEX TERMINAL SETS†

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Two different pursuit–evasion games are considered from the evader's point of view. The phase space is a plane, each of the two players controlling the motion of a point only along its own coordinate. The terminal sets are not convex; in the first problem, the set is an arc of a circle, in the second, the union of tow segments. In both games evasion cannot the achieved by means of programmed controls, but it can be achieved using feedback control. However, the strategies, which are continuous functions of the phase vector, have different properties in each problem. In the first, they cannot guarantee evasion (which is typical for the linear-convex case as well), but in the second they can (which is impossible in linear-convex games with a fixed final time). Verification that evasion is unachievable using such strategies reduces here to proving the solvability of a certain initial-value problem for an advanced differential equation, to which the Schauder principle is applicable.

In many problems in the theory of positional differential games [1–4] evasion cannot be guaranteed using continuous [5] or Carathéodory [6] strategies. This problem has been thoroughly investigated ([5, 5], [1, Sec. 55], [2, Sec. 3], [3, Secs 5, 6, 22, 23]) on the assumption that the dynamics is linear and the terminal set is convex. The properties of strategies which are continuous functions of the phase vector have also been studied without this assumption [7].

The examples presented below demonstrate the essential difference between the case of a non-convex terminal set and the linear-convex case. These examples are constructed making use of only quite simple mathematical tools.

1. EVASION OF AN ARC

Suppose the motion of a point $x = (x_1, x_2)$ in the plane is described by the following system of differential equations

$$\dot{x}_1 = v, \quad \dot{x}_2 = 2(1-t)u, \quad t \in [0, 1] \tag{1.1}$$

where u and v are the controls of the two players, chosen in the interval [0, 1]. We shall use classical motions. The function x: $[0, 1] \to \mathbf{R}^2$ must be absolutely continuous and satisfy the differential equations almost everywhere. The initial position of the point is assumed to be zero:

$$x(0) = 0 \tag{1.2}$$

The terminal set M is defined in the x_1, x_2, plane by the relations $x^2 = 1, x_1 \geqslant 0, x_2 \geqslant 0$ $(x^2 = x_1^2 + x_2^2)$. Thus, M is a quarter of the circumference of the unit circle about the origin. The function $u(t)$ is assumed to be Lebesgue-measurable. It is required to choose the control v in such a way that the terminal set can be evaded at a finite time, i.e. that the condition $x(1) \in M$ is satisfied for any admissible noise $u(t)$.

Before we consider strategies that are continuous with respect to the phase variable, let us convince ourselves that the problem is non-degenerate. To that end, we must verify that programmed controls $v = v(t)$ cannot guarantee evasion of the TM and outline a discontinuous way to construct v that will guarantee evasion.

We will first show that programmed control $v = v(t)$ do not guarantee evasion even if u is taken to be a constant.

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Property 1.1. For any measurable function v: $[0, 1] \to [0, 1]$, a constant $u \in [0, 1]$ exists such that the corresponding solution x of the initial-value problem (1.1), (1.2) satisfies the condition $x(1) \in M$.

Proof. Put

$$\langle v \rangle = \int_0^1 v(\tau)d\tau$$

and suppose $u \equiv \sqrt{1 - \langle v \rangle^2}$. Then $u \in [0, 1]$ and for the solution of problem (1.1), (1.2) we have $x_2(1) = u$. Thus, $x^2(1) = 1, x_1(1) \geqslant 0, x_2(1) \geqslant 0$, and consequently $x(1) \in M$.

We now present a method of feedback control that guarantees evasion. We will construct a simple method, without concerning ourselves about its optimality in any sense.

Property 1.2. For $0 \leqslant t < 1/2$, define $v = 1$, For $1/2 \leqslant t \leqslant 1$, define

$$v = \begin{cases} 1, & \text{if} \quad x_2(1/2) \geq 12/25 \\ 0, & \text{if} \quad x_2(1/2) < 12/25 \end{cases}$$

Then for any measurable function u: $[0, 1] \to [0, 1]$ the corresponding solution of initial-value problem (1.1), (1.2) satisfies the inequality

$$|1 - |x(1)|| > 1/10 \tag{1.3}$$

where the norm is the Euclidean norm.

In fact, it will be shown that this mode of control yields a distance of slightly more than 1/10 from the terminal set.

Proof. If $x_2(1/2) \geqslant 12/25$, then $v = 1$ for all $t \in [0. 1]$, and therefore $x_1(1) = 1$. Since u is non-negative, we have $x_2(1) \geqslant x_2(1/2) \geqslant 12/25$. Thus, $|x(1)|\sqrt{769/25} > 11/10$ and inequality (1.3) holds with something to spare. But if $x_2(1/2) < 12/25$, then $v = 1$ for $t \in [0, 1/2]$ and $v = 0$ for $t \in [1/2, 1]$. Consequently, $x_1(1) = 1/2$. Now

$$x_2(1) = x_2(1/2) + 2\int_{1/2}^{1} (1 - \tau)u(\tau)d\tau < 12/25 + 2\int_{1/2}^{1} (1 - \tau)d\tau = 73/100$$

Therefore, $|x(1)| < \sqrt{7829}/100 < 9/10$. Inequality (1.3) is again satisfied with something to spare.

Remark 1.1. The simple mode of evasion presented in the formulation of Property 1.2 is far from optimal. For example, optimization of the constant 12/25 enables one to improve the guaranteed result slightly.

Remark 1.2. A control law that guarantees evasion in this problem may also be implemented as a positional strategy, following the well-known formalization of [1], using stepwise schemes with meshes in a time interval that become increasingly finer. In fact, the control law in the formulation of Property 1.2 is consistent with any positional strategy of player u, which eliminates the possibility of a guaranteed transfer to the terminal set. It follows by the Alternative Theorem [1, Sec. 17] that a positional strategy of player v exists that guarantees evasion in constructive motions. This conclusion is similar to a previous one ([2, p. 23], [1, p. 243]).

We shall now show that, in this problem, evasion of the terminal set cannot be achieved using strategies $v = v(t, x_1, x_2)$ that satisfy the Carathéodory conditions, that is, such that the function $v(t, x_1, x_2)$ is continuous in x_1, x_2 for almost every fixed t and Lebesgue-measurable in t for all fixed x_1, x_2. For such strategies, the absolutely continuous solution of the initial-value problem (1.1), (1.2) may not be unique. We shall assume that a strategy guarantees evasion if none of the motions corresponding to that strategy and different admissible noises terminate at the terminal set.

The fact that evasion cannot be guaranteed in this problem using strategies that satisfy the Carathéodory conditions is easily deduced from a previous result [7, Corollary 1]. As a target set in the space of the trajectories one takes the collection of all continuous functions that end at the terminal set. It is immediately obvious that all the conditions of [7, Corollary 1] are satisfied; the fact that the relevant intersections are acyclic follows from their convexity, and that they are not empty follows from Property 1.1 established above. This method of proof is based in the final analysis on theorem due to Eilenberg-Montgomery.

However, taking the specific features of the problem into consideration, one can give a direct proof based on Schauder's principle [9, p. 616], without making use of algebraic topology. With this approach, one can actually establish a stronger result for the example under consideration: A Cratheódory strategy cannot guarantee evasion even if the noise u is taken to be independent of the time t. We thus obtain the following property, an analogue of Property 1.1 (which was concerned with the simpler case of programmed controls).

Property 1.3. Suppose the function v: $[0, 1] \times \mathbf{R}^2 \to [0, 1]$ satisfies the Carathéodory conditions. Then a constant $u \in [0.1]$ exists such that the initial-value problem (1.1), (1.2), where $v = v(t, x_1, x_2)$, has at least one solution x for which $x(1) \in M$.

Proof. We have to establish the existence of an absolutely continuous function x: $[0, 1] \to \mathbf{R}^2$ and a number $u \in [0.1]$ such that

$$\dot{x}_1 = v(t, x_1, x_2), \quad \dot{x}_2 = 2(1-t)u, \quad x(0) = 0, \quad x^2(1) = 1$$

Hence it can be seen that $x_2(t) = t(2-t)u$. Substituting this expression into the last of the equalities, we find that $u = \sqrt{1 - x_1^2(1)}$. Eliminating x_2 and u from this system of relations, we obtain the following initial-value problem for a scalar advanced functional-differential equation.

$$\dot{x}_1(t) = v\left(t, x_1(t), t(2-t)\sqrt{1 - x_1^2(1)}\right), \quad x_1(0) = 0$$

It remains to verify that this problem is solvable; this will imply the existence of x and u with the necessary properties.

Let S denote the set of all functions z: $[0, 1] \to [0, 1]$ that satisfy a Lipschitz condition with constant 1. By the Arzela-Ascoli Theorem [9, p. 48], S is a compact set in the space of continuous functions. In addition, the set S is convex. For $z(\cdot) \in S$, $t \in [0, 1]$, we set

$$F(z(\cdot))(t) = \int_0^t v\left(\tau, z(\tau), \tau(2-\tau)\sqrt{1 - z^2(1)}\right) d\tau$$

The function $F(z(\cdot))(t)$, as a function of the argument t, satisfies a Lipschitz condition with constant 1, since $0 \leq v \leq 1$. The mapping F takes S into itself and is continuous in the uniform norm of the set of continuous functions. Bye Schauder's theorem, F has at least one fixed point in S. Thus, the initial-value problem introduced above for the relevant functional-differential equation is solvable.

Remark 1.3. Suppose the strategy $v = v(t, x_1, x_2)$ is such that the solution of initial-value problem (1.1), (1.2) is unique for any number $u \in [0, 1]$ (for example, $v(t, x_1, x_2)$ satisfies a Lipschitz condition). One can then replace Schauder's principle in the proof of Property 1.3 by the fact that a continuous scalar function defined in a closed interval and changing sign there must vanish somewhere. One can then approximate an arbitrary Carathéodory strategy by a Lipschitz strategy.

Remark 1.4. One can consider in a analogous way games for (1.1), (1.2) with other curves as the terminal set M. For example, M may be a segment of a straight line (this gives the convex case) or an "inverted" quarter-circle, that is, the set defined by

$$(x_1 - 1)^2 + (x_2 - 1)^2 = 1, \quad x_1 \leq 1, \quad x_2 \leq 1$$

Remark 1.5. Properties like those of the example analysed above may also be found in some degenerate situations. The following obvious reasoning makes it possible, given a differential game, to formally construct an equivalent game with non-convex terminal set. For a given initial position and the usual conditions on the dynamics of the system, all trajectories of the game will remain in a certain closed sphere in phase space. We can assume that the terminal set is contained in that sphere Adding a point outside the sphere to the terminal set makes it non-convex and non-connected, while all the main properties of the game obviously remain the same. Under these conditions the part of the terminal set that actually participates in the differential game is unchanged.

Remark 1.6. There is a simple example of a game that does not use differential equations, for which the situation described in this section nevertheless arises. Suppose the first player chooses a number $u \in [0, 1]$ and the second chooses a number $v \in [0, 1]$. The aim of the second player is to maximize the payoff, which is equal to $|u - v|$. Obviously, no choice of a constant v will guarantee a non-zero payoff (the analogue of programmed controls in differential games). Suppose now that the second player designates his number as $v = \varphi(u)$, where φ: $[0, 1] \to [0, 1]$ is a fixed function. No continuous function φ will guarantee a non-zero result for the second player (the analogue of continuous strategies in differential games). Indeed, the function φ has a fixed point, as follows easily, e.g., from Schauder's principle. On the other hand, the discontinuous function

$$\varphi(u) = \begin{cases} 1, & 0 \le u \le 1/2 \\ 0, & 1/2 < u \le 1 \end{cases}$$

guarantees the second player a payoff of at least $^1/_2$ (the analogue of discontinuous strategies).

2. EVASION OF TWO SEGMENTS

Consider a point with coordinates x_1, x_2 in the plane, whose motion is described by the system

$$\dot{x}_1 = u, \quad \dot{x}_2 = v \tag{2.1}$$

and which begins at zero:

$$x(0) = 0 \tag{2.2}$$

The independent variable $t \in [0, 2]$ is time. The controls u and v of the two players lie in $[-1, 1]$. The solution x: $[0, 1] \to \mathbf{R}^2$ is absolutely continuous and must satisfy the system of differential equations almost everywhere. We assume that the function $u = u(t)$ is Lebesgue-measurable. The terminal set M is the union of the segment $x_1 = 1, 0 \leqslant x_2 \leqslant 1$ with the segment $x_1 = -1, -1 \leqslant x_2 \leqslant 0$. The problem is, using the control v, to guarantee evasion at a finite instant of time, $x(1) \notin M$, in the presence of noise $u(t)$.

We shall verify that programmed controls do not guarantee evasion in this problem, even if attention is confined to just two possible noises.

Property 2.1. For any measurable function $v = v(t)$, where v: $[0, 1] \to [-1, 1]$, at least one of the two solutions of problem (2.1), (2.2) corresponding to the constants $u \equiv 1$, $u \equiv -1$ satisfies the relation $x(1) \in M$.

Proof. Any solution of problem (2.1), (2.2) is such that $|x_2(1)| \leqslant 1$, since $|v(t)| \leqslant 1$. If $\langle v \rangle \geqslant 0$, we need only take $u \equiv 1$. But if $\langle v \rangle \leqslant 0$, we take $u \equiv -1$. When $\langle v \rangle = 0$, one can take either of these two noise values.

We will now establish properties that unable us to construct a strategy which guarantees evasion of the terminal set. In particular, the strategies constructed will be continuous.

Property 2.2. Let v: $[0, 1] \times \mathbf{R} \to [-1, 1]$; assume that the functions $v(t, t)$, $v(t, -t)$ of the argument t and also the function u: $[0, 1] \to [-1, 1]$ are measurable, and moreover

$$\langle v(\tau, \tau) \rangle < 0, \quad \langle v(\tau, -\tau) \rangle > 0$$

where the integration is performed with respect to τ. Then the problem

$$\dot{x}_1 = u(t), \quad \dot{x}_2 = v(t, x_1), \quad x(0) = 0, \quad x(1) \in M \tag{2.3}$$

has no solutions.

Proof (by *reductio ad absurdum*). Suppose problem (2.3) has a solution x. It follows from the condition $x(1) \in M$ that $|x_1(1)| = 1$. Let us first consider the case $x_1(1) = 1$. Since $|u(t)| \leqslant 1$, this is possible only if $u(t) = 1$ almost everywhere. Then $x_1(t) = t$ for all $t \in [0. 1]$, and therefore $x_2(1) = \langle v(\tau, \tau) \rangle < 0$. Thus, $x(1) \notin M$. Similarly, when $x_1(1) = -1$ we have $u(t) \equiv -1$, $x_1(t) \equiv -t$, $x_2(1) > 0$, and again $x(1) \notin M$. This contradiction shows that problem (2.3) is unsolvable.

Let us assume that the function $v = v(t, x_1)$ satisfies the assumptions of Property 2.2. Suppose moreover that this function, paired with any admissible noise $u = u(t)$, yields an initial-value problem (2.1), (2.2) which has an absolutely continuous solution. (For that to be true it will suffice that for any continuous ψ the superposition $v(t, \psi(t))$ should be measurable.) Then the function v can serve as a strategy guaranteeing evasion in the differential game under consideration in classical motions. An example is the continuous strategy

$$v(x_1) = \begin{cases} 1, & x_1 \le -1 \\ -x_1, & |x_1| \le 1 \\ -1, & x_1 \ge 1 \end{cases}$$

As shown previously, this way of constructing evasion strategies in this game is based on very simple premises, reducing to the consideration of only two noise values. However, it does not yield a lower estimate of the distance $\rho(x(1), M)$ from the point $x(1)$ to the terminal set M. We shall now present a method that yields such an estimate.

Property 2.3. Let $\varepsilon \in [0, 1/2]$ be a fixed number and let $v = v(x_1)$ be a function with values in $[-1, 1]$ such that

$$v(x_1) = -\operatorname{sgn} x_1 \quad \text{for} \quad |x_1| > \varepsilon \tag{2.4}$$

Suppose the function u: $[0, 1] \to [-1, 1]$ is measurable and the function x: $[0, 1] \to \mathbf{R}^2$ is absolutely continuous and satisfies the initial condition

$$\dot{x}_1 = u(t), \quad \dot{x}_2 = v(x_1), \quad x(0) = 0 \tag{2.5}$$

Then

$$\rho(x(1), M) \geq \varepsilon_1, \quad \varepsilon_1 = (1-2\varepsilon)/\sqrt{5} \tag{2.6}$$

Varying ε, one can choose functions u and v such that equality is achieved in this estimate.

Note that, for the estimate in Property 2.3 to be true, the behaviour of the function $v(x_1) \in [-1, 1]$ for $|x_1| \leq \varepsilon$ is immaterial. For these x_1 values, the function may be discontinuous or even many-valued. It is only important that problem (2.5) should have an absolutely continuous solution.

Proof. If $|x_1(1)| \leq \varepsilon$, then $\rho(x(1), M) \geq 1 - \varepsilon \geq 1/2 > 1/\sqrt{5}$, and inequality (2.6) holds. Now let $|x_1(1)| > \varepsilon$. To fix our ideas, let us assume that $x_1(1) > \varepsilon$. Consideration of the case of negative $x_1(1)$ is analogous. Let ρ_+ denote the distance from $x_1(1)$ to the segment $x_1 = 1, 0 \leq x_1 \leq 1$, and ρ_- the distance from the same point to the segment $x_1 = -1, -1 \leq x_2 \leq 0$. Then $\rho(x(1), M) = \min(\rho_-, \rho_+)$, and moreover

$$\rho_- \geq 1 + x_1(1) > 1 > \varepsilon_1$$

To establish inequality (2.6), it remains to show that ρ_+ may also be estimated from below by the same number. A number $\xi \in [0, 1)$ exists such that $x_1(\xi) = \varepsilon$, and for $t > \xi$ we have $x_1(t) > \varepsilon$. We have $\varepsilon \leq \xi$. Indeed

$$\varepsilon = x_1(\xi) = x_1(0) + \int_0^\xi u(\tau)d\tau \leq \xi$$

In addition

$$x_1(1) = x_1(\xi) + \int_\xi^1 u(\tau)d\tau \leq \varepsilon + 1 - \xi$$

Furthermore

$$x_2(1) = x_2(0) + \int_0^\xi v(x_1(\tau))d\tau + \int_\xi^1 (-1)d\tau \leq -1 + 2\xi$$

Thus, we have inequalities

$$1 - x_1(1) \geq \xi - \varepsilon \geq 0, \quad -x_2(1) \geq 1 - 2\xi \tag{2.7}$$

Let us first assume that $\xi \leq 1/2$. It then follows from the second inequality of (2.7) that $x_1(1) \leq 0$. Therefore

$$\rho_+ = \sqrt{(1 - x_1(1))^2 + x_2^2(1)}$$

Hence, using inequalities (2.7) and the inequality $\xi \leq 1/2$, we obtain

$$\rho_+ \geq \sqrt{(\xi - \varepsilon)^2 + (1 - 2\xi)^2} = \sqrt{5(\xi - (2+\varepsilon)/5)^2 + \varepsilon_1^2} \geq \varepsilon_1$$

Now consider the case $\xi > 1/2$. Using the first inequality of (2.7), we obtain

$$\rho_+ \geq 1 - x_1(1) \geq \xi - \varepsilon > 1/2 - \varepsilon \geq \varepsilon_1$$

The required estimate is thus proved in all cases.

It remains to verify that there are motions for which the estimate is an equality. To that end, varying the number ε, we construct functions u and v which make it an equality. Define

$$u(t) = \begin{cases} 5 - 2/\varepsilon_2, & 0 \le t \le \varepsilon_2 \\ 1, & \varepsilon_2 < t \le 1; \quad \varepsilon_2 = (2+\varepsilon)/5 \end{cases}$$

Since $\varepsilon \in [0, 1/2]$, we have $5 - 2/\varepsilon_2 \in [0, 1]$. Integrating, we find that

$$x_1(t) = \begin{cases} (5 - 2/\varepsilon_2)t, & 0 \le t \le \varepsilon_2 \\ t - \varepsilon_2 + \varepsilon, & \varepsilon_2 \le t \le 1 \end{cases} \tag{2.8}$$

Therefore, $x_1(1) = (3 + 4\varepsilon)/5 \in [3/5, 1]$. Let

$$v(x_1) = \begin{cases} 1, & x_1 \le \varepsilon \\ -1, & x_1 > \varepsilon \end{cases} \tag{2.9}$$

Condition (2.4) is satisfied. Substituting (2.8) into (2.9), we get

$$v(x_1(t)) = \begin{cases} 1, & 0 \le t \le \varepsilon_2 \\ -1, & \varepsilon_2 < t \le 1 \end{cases}$$

Integration gives

$$x_2(t) = \begin{cases} t, & 0 \le t \le \varepsilon_2 \\ 2\varepsilon_2 - t, & \varepsilon_2 \le t \le 1 \end{cases}$$

Thus, $x_2(1) = (-1 + 2\varepsilon)/5 \in [-1/5, 0]$. Taking the limiting values $x_1(1)$ and $x_2(1)$ into consideration, we see that the distance $\rho(x(1), M)$ is obtained at the point $(1, 0)$. Consequently

$$\rho(x(1), M) = \sqrt{(1 - x_1(1))^2 + x_2^2(1)} = \frac{1}{5}\sqrt{(4\varepsilon - 2)^2 + (2\varepsilon - 1)^2} = \varepsilon_1$$

which makes the estimate an equality.

Remark 2.1. Let $\varepsilon \in (0, 1/2)$. There are many ways to complete the definition of the function (2.4) for $|x_1| \le \varepsilon$ so as to obtain a continuous function with values in $[-1, 1]$. By Property 2.3, the continuous strategy thus obtained guarantees evasion of the terminal set in classical motions. For example, define

$$v(x_1) = \begin{cases} 1, & x_1 < -\varepsilon \\ -\varepsilon^{-1}x_1, & |x_1| \le \varepsilon \\ -1, & x_1 > \varepsilon \end{cases}$$

One can also complete the definition of the function (2.4) as an arbitrarily smooth function, even an infinitely differentiable one

$$v(x_1) = \begin{cases} 1, & x_1 \le -\varepsilon \\ 1 - 2\exp(-(x_1 + \varepsilon)^{-2})\exp(-(x_1 - \varepsilon)^{-2})), & |x_1| < \varepsilon \\ -1, & x_1 \ge \varepsilon \end{cases}$$

Remark 2.2. When $\varepsilon = 0$ the function (2.4), with its definition completed at $x_1 = 0$ by an arbitrary number in $[-1, 1]$, yields a discontinuous strategy. Since this discontinuous strategy and initial-value problem (2.5) are fairly simple in structure, one can consider absolutely continuous solutions of problem (2.5). By Property 2.3, the strategy introduced here guarantees evasion of the terminal set by the point at a finite instant of time, by a distance of at least $1/\sqrt{5} = 0.447\ldots$.

Remark 2.3. When $\varepsilon = 0$, the function (2.4) can be redefined at $x_1 = 0$ as a multi-valued mapping, by letting $v(0)$ be the entire segment $[-1, 1]$. We obtain a multi-valued strategy which is upper semicontinuous (with respect to inclusion). The initial-value problem (2.5) will then involve a differential inclusion. By Property 2.3, this multi-valued strategy guarantees evasion of the terminal set at a finite instant of time. It also guarantees a distance of at least $1/\sqrt{5}$ from the terminal set, as in the case of the discontinuous single-valued strategy defined above.

Remark 2.4. The evasion problem studied in this section demonstrates, in particular, that under the assumptions of Corollary 1 of [7] and hence under the assumptions of the theorem of [7], one cannot drop the assumption that the corresponding intersections are acyclic. Indeed, the problem in question satisfies all the assumptions of the corollary with the exception of acyclicity, but the conclusion of the corollary does not hold here, since in our problem, by virtue of Remark 2.1, one can guarantee evasion using a continuous strategy.

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